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Two separate objectives were stated at the beginning of this research project:

1) To develop new numerical algorithms for the efficient and accurate solution of the continuum equations of viscous motion for high-temperature, chemically non-equilibrium, radiating hypersonic flow.
2) To develop a new non-linear stress strain tensor for continuum equations of motion at high altitudes that is more accurate than the Navier-Stokes equations.

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The bottom line of this final report is that, relative to each of these two objectives, much more has been accomplished than anticipated. For example, in regard to (1), a complete code was developed for computing hard-body flow-field radiation from Navier-Stokes equations taking into account thermodynamic, chemical, and ionization nonequilibrium; and in in regard to (2), hypersonic solutions to the Burnett equations were obtained for the first time, and shown to provide both the non-linear stress-strain tensor and heat flux vector needed to yield computations at high altitudes that are much more accurate than the Navier-Stokes equations. This latter development reverses a commonly accepted opinion of thirty years that the Burnett equations can not be used for such purposes.

The main output of our research is represented by the technical publications listed subsequently for each of the above two areas of research. These combined works are far too lengthy for reiteration of the various technical details here. Consequently, this final report attempts to set in perspective, and to outline in somewhat of an executive-summary type fashion, the salient accomplishments of the subject research, their significance, and potential technical applications. Thus, attention herein is given to technical advances made, rather than to details of how they were made.

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Department of AERONAUTICS and ASTRONAUTICS
STANFORD UNIVERSITY

FINAL REPORT
For Period 1 September 1986 to 31 December 1989

IMPROVED COMPUTATIONAL FLUID DYNAMICS FOR CONTINUUM HYPERSONIC FLOW

A.R.O. CONTRACT DAAL 03-36-K-0139

Professor Dean R. Chapman
Professor Robert W. MacCormack

Department of Aeronautics and Astronautics
Stanford, CA 94305

January 1990

FINAL REPORT FOR ARO CONTRACT DAAL03-36-K-0139
1 September 1986 -- December 31, 1989

IMPROVED COMPUTATIONAL FLUID DYNAMICS FOR CONTINUUM HYPERSONIC FLOW

Dean R. Chapman and Robert W. MacCormack

RESEARCH OBJECTIVES

Two separate objectives were stated at the beginning of this research project:

(1) To develop new numerical algorithms for the efficient and accurate solution of the continuum equations of viscous motion for high-temperature, chemically non-equilibrium, radiating hypersonic flow.

(2) To develop a new non-linear stress strain tensor for continuum equations of motion at high altitudes that is more accurate than the Navier-Stokes equations.

The bottom line of this final report is that, relative to each of these two objectives, much more has been accomplished than anticipated. For example, in regard to (1), a complete code was developed for computing hard-body flow-field radiation from Navier-Stokes equations taking into account thermodynamic, chemical, and ionization nonequilibrium; and in in regard to (2), hypersonic solutions to the Burnett equations were obtained for the first time, and shown to provide both the non-linear stress-strain tensor and heat flux vector needed to yield computations at high altitudes that are much more accurate than the Navier-Stokes equations. This latter development reverses a commonly accepted opinion of thirty years that the Burnett equations can not be used for such purposes.

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(1) NEW NUMERICAL ALGORITHMS AND CFD CODES

This area of research, supervised primarily by Professor MacCormack, provided support for two research assistants whose thesis topics contributed directly to the research program: Graham Candler who received a Ph.D. degree in June 1988, and Tahir Gökçen who received a Ph.D. degree in June 1989. The relevant research publications are listed below in chronological order.

1A. Gökçen, T., MacCormack, R.W. and Chapman, D.R., *"Computational Fluid Dynamics Near the Continuum Limit'"*, AIAA Paper 87-1115, Proceeding of the 8th CFD Conference, Honolulu, June 9-10, 1987, pp. 153-158.

1B. Candler, G.V. and MacCormack, R.W. *"The Computation of Hypersonic Flows in Chemical and Thermal Nonequilibrium"*, Third National Aerospace Plane Technology Symposium, NASA Ames, Moffett Field, CA June 2-4, 1987.

1C. Candler, G.V.. and MacCormack, R.W., *"The Computation of Hypersonic Ionized Flows in Chemical and Thermal Nonequilibrium"*, AIAA paper 88-0511, 26th Aerospace Sciences Meeting, Reno, January 11-14, 1988.

1D. MacCormack, R.W. and Candler, G.V., *"A Numerical Method for Predicting Hypersonic Flow Fields"*, SPIE paper 879-23, Symposium on Innovative Sciences and Technology, Los Angeles, January 10-15, 1988

1E. Candler, G.V. and Park, C., *"The Computation of Radiation from Nonequilibrium Hypersonic Flows"*, AIAA Paper 88-2678, Thermodynamics Plasmadynamics, and Lasers Conference, San Antonio, June 27-29, 1988.

1F. Gökçen, T. MacCormack, R.W. (1989) *"Nonequilibrium Effects for Hypersonic Transitional Flows Using Continuum Approach"*, AIAA paper 89-0461.

This research progressed in stages of incremental addition of physical phenomena to computational codes, combined with development of the associated numerical methods for computing these phenomena. The work involving Candler began with the treatment of thermal/chemical nonequilibrium (1B), then was extended to ionization nonequilibrium (1C, 1D), and finally to the computation of hard-body radiation (1E). It considers 5 chemical species, N_2, O_2, NO, N, O plus electrons/ions, treats shock waves as discontinuous jumps, employs two temperatures —

translational and vibrational — and uses the Navier-Stokes equations of motion. The primary accomplishment of this work is that it has provided for the first time a complete Navier-Stokes code for computing hard-body radiation at every point in a two-dimensional flow field. Being the first such code, it naturally treats certain technical matters in ways that need augmentation or improvement, especially for high altitude computations. Some of these directions for improvement were investigated by T. Gökçen, and others by K. Fiscko and F. Lumpkin, as outlined in the paragraphs which follow.

The research involving Gökçen first developed a new set of surface boundary conditions (1A) for high-altitude flow-field computation. The conventional set of Maxwell surface-velocity slip and Smoluchowski temperature jump are of first-order accuracy only, as are the Navier-Stokes equations. They are applicable for the initial departures from no-slip boundary conditions, but yield sizable errors for flow conditions approaching the free molecule domain. Gökçen's improved boundary conditions reduce to the Maxwell-Smoluchowski ones in the limit of small slip and temperature jump, and also yield the correct free-molecule boundary conditions in the opposite limit of extremely rarefied flow. This is regarded as a significant advance in CFD techniques for hypersonic high-altitude flow simulation.

Gökçen's research then proceeded (1) to develop adaptive grids for treating shock-waves as high-gradient zones of finite thickness within which grid points are automatically clustered, and (2) to include three temperatures (rotational as well as translational and vibrational). The same chemical species were considered as in the Candler-MacCormack work, and the Navier-Stokes equations were also employed. This work represents the beginning of some of the extensions of that work which will be needed to adequately compute hard-body radiation at high altitudes. Further extensions believed necessary will require the introduction of more realistic relaxation models than the simple ones Gökçen and Candler employed (e.g. the rotational relaxation model of Lumpkin discussed below), and the use of more realistic equations of motion than Navier-Stokes (e.g., the Burnett equations, or some adequate approximation thereof).

(2) CONTINUUM EQUATIONS BEYOND NAVIER-STOKES

This area of research, supervised primarily by Professor Chapman, also provided support for two research assistants whose thesis topics contributed directly to the research program: Kurt Fiscko, who received a Ph.D. degree in September 1988, and Forrest Lumpkin, who will receive a Ph.D. degree early in 1990. Relevant publications for this area of research are listed below in chronological order.

2A. Chapman, D.R., Fiscko, K.A., and Lumpkin, F.E., *"Fundamental Problem in Computing Radiating Flow Fields with Thick Shock Waves "*, SPIE Symposium on Innovative Science and Technology, January 10-15, 1988, SPIE Vol 879, pp. 106-112.

2B. Fiscko, K.A. and Chapman D.R., *"Comparison of Shock Structure Solutions Using Independent Continuum and Kinetic Theory Approaches"*, SPIE Symposium on Innovative Science and Technology, January 10-15, 1988, SPIE Vol. 879, pp. 113-122.

2C. Fiscko, K.A. and Chapman, D.R., *"Hypersonic Shock Structure with Burnett Terms in the Viscous Stress and Heat Flux "*, AIAA paper 88-2733, Thermodynamics, Plasmadynamics, and Lasers Conference, San Antonio, June 27-29, 1988.

2D. Fiscko, K.A. and Chapman, D.R., *"Comparison of Burnett, Super-Burnett, and Monte Carlo Solutions for Hypersonic Shock Structure"*, 16th International Symposium on Rarefied Gas Dynamics, Pasadena, July 11-15, 1988.

2E. Lumpkin, F.E., Chapman, D.R., and Park, C. (1989) *"New Rotational Relaxation Model for Use in Hypersonic Computational Fluid Dynamics "*, AIAA paper 89-1737.

This research likewise proceeded in stages of incremental addition of relevant physical phenomena combined with development of associated computational methods. At the outset it was recognized that the existence of thick shock waves at high altitudes precluded the use of the Navier-Stokes equations for realistic flow computations. It has been known for about 25 years that these equations are inaccurate for computing through the structure of a hypersonic shock wave. Our first exploratory work investigated the possibility of adding bulk viscosity to the Navier-Stokes equations in an effort to improve the computations. This yielded realistic shock thicknesses, but not temperature profiles (2A). Use of the Burnett

equations, however, was found to yield realistic temperature profiles as well as shock thicknesses (2B, 2C, 2D). at least for monatomic gases.

Since air is composed dominantly of diatomic gases possessing internal molecular energy of rotation and vibration, as well as translation, the next step was to take into account nonequilibrium rotational energy. Vibrational energy does not affect shock structure appreciably below about Mach 11, the upper limit of present experimental data on shock-wave thickness in nitrogen. Hence it was possible to test computational methods for treating rotational-energy nonequilibrium by comparison with existing shock-wave data. We found that a rotational relaxation method developed for such computation, when used in combination with the Burnett equations, yields results for nitrogen in very close agreement with experimental measurements of shock wave thickness (2E).

Lumpkin also developed an approximate truncated form of the Burnett equations yielding density and temperature profiles in hypersonic shock structure that are quite close to results from the full Burnett equations. The truncated form is simpler, requires less computer time, and is not expected to add any significant numerical stability problems beyond those of the conventional Navier-Stokes equations.

Our primary conclusion from this area of research, therefore, is that the Burnett equations, rather than the Navier-Stokes equations, should be used for the hydrodynamic portion of a code designed to compute hard-body flow-field radiation from vehicles flying at high altitude (for example, at altitudes above about 50 km for a body with nose radius of 10 cm, or above about 65 km for a nose radius of 1 m). This conclusion has been validated for 1D flow by a comparison of computational and experimental structure of hypersonic shock waves.

Historical Perspective. - Our main conclusion concerning the Burnett equations has formed quite a surprise, even somewhat of a shock, to many scientists who have worked in the field of high temperature hypersonic gas dynamics. For about 30 years the Burnett equations were generally *rejected* as a means of describing hypersonic flows that are far out of thermodynamic equilibrium. Some example statements clearly illustrating this rejection, taken from scientific publications between 1959 and 1989, are attached to this report (Attachment 1). Their sources are listed in Attachment 1a. A main contributing reason for the rejection was that, prior to our work, no one had been able to obtain even a single solution, either numerical or analytical, for hypersonic shock wave structure from the Burnett equations, in spite of many attempts. This led to the wide spread view that there must be something fundamentally wrong with the Burnett equations. Actually, we found that the problem was due to inadequate mathematical methods previously used, rather than to the

Burnett equations. The main conclusion of our work is that the Burnett equations are of acceptable engineering accuracy for computing flow conditions in shock wave structures that are far out of thermodynamic equilibrium due to the nonequilibrium state of molecular translational velocities. The study of monatomic gases was ideal for rigorously making such an assessment, because they possess only molecular translational energy.

It is to be noted that there were plausible reasons for rejecting the Burnett equations during the past three decades. First, the basic series expansion method of deriving from Boltzmann's equation first the equations of Navier-Stokes, then Burnett, Super-Burnett, etc., has long been known to yield only an asymptotically convergent series. With such series there is always a maximum number of expansion terms giving improved approximation; any further increase does not improve, but makes the approximation worse. Second, despite numerous attempts over these decades, it was not possible to obtain any solution to the Burnett equations for strong shock waves. Thus, knowing (1) that the basic series method must go bad if the expansion order is too high, and (2) that solutions were unobtainable from the Burnett equations (second order expansion), it was not unreasonable to believe that the series expansion method went bad after the first-order Navier-Stokes terms, and hence to reject the Burnett equations. It now appears from our research, which has shown the third-order Super-Burnett equations for Maxwellian gas to yield worse shock structure results than the Burnett equations, that the basic expansion method gets worse after the second-order Burnett terms, rather than after the first-order Navier-Stokes terms.

<u>Progress During Past Six Months.</u> - During the latter portion of this contract we have begun the basic mathematical analysis for the numerical computation of 2D flows using the Burnett equations. One or our graduate students, Xiaolin Zhong, under the primary supervision of Professor MacCormack, has pointed out a fundamental mathematical problem encountered in such computations, and has devised a solution to this problem. As computations are attempted at progressively higher altitudes, solutions to the Burnett equations eventually become unstable (at altitudes above that at which the grid mesh spacing becomes less than the order of one mean free path). In this sense, the statements listed in the attachment are not totally incorrect. Zhong's solution is to add to the Burnett equations just a few carefully selected terms of the higher-order Super-Burnett equations. The resulting equations, termed the "augmented Burnett" equations, have been shown to be analytically well posed and to retain the accuracy advantage that the Burnett equations have over the Navier-Stokes equations for hypersonic conditions. Numerical

computations have shown that the added terms stabilize the 1D Burnett equations even for extremely small mesh spacing. A test of the stability of 2D augmented Burnett equations, started only recently, was not completed by the termination date of the contract (December 31, 1989). One result obtained, however, is that the computer time required to numerically solve the 2D Burnett equations is only about 40% more than that required for the 2D Navier-Stokes equations.

An abstract of this most recent work of Zhong (Attachment 2) was submitted November 10, 1989, as a proposed paper for presentation at the AIAA/ASME 5th Thermophysics and Heat Transfer Conference, June 18-20, 1990 in Seattle, Washington.

In addition to the five graduate student research assistants mentioned above, several others were supported for short periods of time in connection with special portions of the overall research project.

POTENTIAL TECHNICAL APPLICATIONS

Future Improvements in Computation of Hard-Body Radiation at High Altitudes

Our comments on future improvements that can be made in the technology of computing high-altitude flow-field radiation pertain only to the hydrodynamic portion of an overall computational code. It is recognized that significant improvements by others in, for example, the molecular radiation/absorption portion of such a code may also be possible.

Relative to the hard-body radiation code that Candler and Park developed (1E) for relatively low altitudes, accurate computations at high altitudes will require the following modifications:

(a) Computation through the finite thickness of hypersonic shock-wave structure, rather than treating the shock as a discontinuity.

(b) Computation of non-equilibrium rotational temperature, rather than assuming it is equal to translational temperature

(c) Employment of Burnett equations, or an approximate form thereof, rather than the Navier-Stokes equations.

Modification (a) is necessary since shock-wave thickness at high altitude can be greater than the normal detachment distance, and can represent the primary zone producing flow-field radiation; (b) is required since nonequilibrium of rotational temperature produces an overshoot in the translational temperature which affects radiation; and (c) is required in order for the code to yield realistic profiles of temperature and density in the shock structure. Modifications (a) and (b) are comparatively straight forward, and not anticipated to cause any significant problem. Modification (c), however, is in a completely different category. No attempt has yet been made to obtain numerical solutions of the Burnett equations for hypersonic 2D flow over a body of revolution. As noted above, the numerical stability problem with the Burnett equations is expected to arise for altitudes above that at which the grid mesh spacing is less than the order of a mean-free path length. Our augmented Burnett equations hopefully will solve this problem, but time did not permit a demonstration or test of that as yet.

Attachment 1

THREE DECADES OF REJECTION

(1959) THE BURNETT EQUATIONS MAKE NO IMPROVEMENT WHICH MERITS THE TROUBLE OF SOLVING THEM. FOR A SHOCK MACH NUMBER OF MORE THAN 2.5, IT REMAINS FUNDAMENTALLY DOUBTFUL THAT ANY OF THESE (BURNETT) THEORIES CAN BE CORRECT.

(1969) THE BURNETT EQUATIONS HAVE BEEN CONSIDERED TO BE OF NO PRACTICAL INTEREST.

(1976) THIS RESULT TENDS TO CONFIRM THE BELIEF THAT THE BURNETT EQUATIONS CANNOT BE USED TO PROGRESS INTO RAREFIED REGIONS WHERE THE NAVIER-STOKES EQUATIONS ARE ALREADY INVALID.

(1983) THE BURNETT EQUATIONS HAVE NEVER ACHIEVED ANY NOTABLE SUCCESS IN DESCRIBING DEPARTURES FROM THE NAVIER-STOKES MODEL. THEIR PRACTICAL IMPORTANCE IS NEGLIGIBLE.

(1988) THE FAILURE OF THESE EXPANSIONS (E.G. BURNETT) TO HANDLE PROBLEMS INVOLVING SIGNIFICANT DEPARTURES FROM EQUILIBRIUM, SUCH AS IN STRONG SHOCK WAVES, HAS LED TO A CERTAIN DISENCHANTMENT WITH ALL SERIES METHODS IN RECENT DECADES.

(1989) THE BURNETT EQUATIONS HAVE NOT DEMONSTRATED A SUPERIORITY OVER NAVIER-STOKES EQUATIONS IN TREATING STRONGLY NON-LINEAR PROCESSES SUCH AS SHOCKS.

EXAMPLE REFERENCES REJECTING BURNETT EQUATIONS

Talbot, L. and F.S. Sherman, "Structure of Weak Shock Waves in a Monatomic Gas", NASA Memorandum, 12-14-58, 1959.

Kogan, M.N., "Rarified Gas Dynamics", Plenum Press, New York, 1969.

Tannehill, J.C. and G.R. Eisler, "Numerical Computation of the Hypersonic Leading Edge Problem Using the Burnett Equations", The Physics of Fluids, Vol. 19, No. 1, p. 9-15, 1976.

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Narasimha, R. and P. Das, "A Spectral Solution of the Boltzman Equation for the Infinitely Strong Shock," National Aeronautical Laboratory (India) Technical Memorandum DU 8802, October 1988.

McLennan, J.A. "Introduction to Non-Equilibrium Statistical Mechanics", Prentice Hall, 1989.

Extended Abstract submitted November 10, 1989 to AIAA/ASME 5th Thermophysics and Heat Transfer Conference, Seattle, Washington, June 18 - 20, 1990

Stability Analysis and Augmentation of the Burnett Equations

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Abstract

Although the Burnett equations have provided much greater accuracy than the Navier-Stokes equations for high-altitude hypersonic CFD, they are unstable to very small wave-length disturbances encountered in fine-mesh solutions. A stability analysis for 1-D flow is used to develop "augmented Burnett equations" which are stable; and which yield essentially the same results for hypersonic shock wave structure as do the Burnett equations. Research on 2-D flows is underway and some results are anticipated to be available prior to the meeting and preprint preparation.

1 Background

Future hypersonic vehicles, such as AOTV, will operate at very high altitude, where shock wave thickness can be comparable to or greater than the detachment distance. Since the Navier-Stokes equations are inaccurate for computing through hypersonic shock structure, some other set of continuum equations is required for CFD applications at these altitudes. One possible set is the Burnett equations.

The validity of the Burnett equations has been the subject of debates since their derivation by D. Burnett [3] in 1935. Since the convergence of the series expansion employed in the solution method has never been proved, great difficulty has been encountered in applying the equations to practical problems, and many authors have questioned the usefulness of these equations [4].

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‡Professor, Fellow AIAA

In spite of the uncertainty, extensive effort has been made to solve the Burnett equations. Wang Chang in 1948 [5], Zoller in 1951 [14], and later Talbot and Sherman in 1959 [12] attempted to solve the equations for the shock wave structure problems using different methods. However, they could only obtain shock wave solutions for $M < 1.9$. In 1976 Simon and Foch integrated the equations to $M = 4$. Fiscko and Chapman [6, 7] recently have integrated the equations for hard-sphere gas model and argon to $M \approx 50$, but could not obtain solutions for Maxwellian gas model above $M \approx 3.8$. In all these investigations, it was found that whenever the Burnett solutions are available, they are in much closer agreement with experimental results or Direct Simulation Monte Carlo (DSMC) solutions than the Navier-Stokes solutions, but, as the Mach numbers increase or the computational grid meshes are refined (for strong shock), the computations become less stable, or unstable.

The stability property of the Burnett equations was first quantitatively studied by Bobylev in 1982 [2]. He found that the solutions of the equations are exponentially unstable for small periodic perturbations when the wave length is smaller than some critical value. The super-Burnett equations also leads to the same conclusion. In order to stabilize the Burnett equations without sacrificing accuracy, a set of augmented Burnett equations is proposed in this paper.

2 Approach

The constitutive equations of the gas flow can be derived by the Chapman-Enskog method, which is a successive approximation method to solve the Boltzmann equations:

$$\begin{cases} \sigma = \sigma^{(1)} + \sigma^{(2)} + \sigma^{(3)} + \cdots \\ q = q^{(1)} + q^{(2)} + q^{(3)} + \cdots \end{cases} \tag{1}$$

Where σ and q are viscous stress and heat flux respectively, and $\sigma^{(1)}$ & $q^{(1)}$ are the first order approximations, i. e. the Navier-Stokes equations; $\sigma^{(2)}$ & $q^{(2)}$ the second order approximations, i. e. the Burnett equations; $\sigma^{(3)}$ & $q^{(3)}$ the super-Burnett equations, and so on.

Guided by the stability analysis, we obtained the augmented Burnett equations by keeping all terms in equation (1) up to second order, then augmenting them with a few higher order terms selected form the super Burnett equations:

$$\begin{cases} \sigma = \sigma^{(1)} + \sigma^{(2)} + \sigma_a \\ q = q^{(1)} + q^{(2)} + q_a \end{cases} \tag{2}$$

Where, the augmented terms are as follows:

$$\left\{ \begin{aligned} \sigma_a &= \frac{\mu^3}{p^2}\{\omega_7 RT\, u_{xxx}\} \\ q_a &= \frac{\mu^3}{p\rho}\{\theta_7 R\, T_{xxx} + \theta_6(\frac{RT}{\rho})\, \rho_{xxx}\} \end{aligned} \right. \tag{3}$$

The numerical values of the coefficients of the equations (3) are chosen so that the equations (2) and equations (3) meet the following requirements:

1. Be stable by the linearized stability analysis;
2. Keep the accuracy of the equations up to the Burnett order.

The constants for one-dimensional cases for Maxwellian gas are $\omega_7 = \frac{2}{9}$, $\theta_6 = -\frac{5}{8}$, and $\theta_7 = \frac{11}{16}$.

3 Results

The new set of equations is analyzed and tested by the following problems and applications.

<u>The Stability Analysis</u>

The characteristic trajectory of the linearized augmented Burnett equations is obtained by using the same method of Bobylev. It is shown that the new equations are stable against perturbations of arbitrarily small wave length.

<u>The Forced Plane Ultrasonic Wave Propagation</u>

The Monatomic gas sound attenuation and dispersion results of the new equations are obtained for the full flow regime ranging from continuum to free molecular limit. It is found that, in the transition regimes, the results of the augmented Burnett equations and the Burnett equations agree much better with those of the experiments than do the results of the Navier-Stokes equations and the super Burnett equations (Fig. 1).

<u>The Shock Wave Structure</u>

The shock profiles of the augmented Burnett equations at Mach number from 1 to 30 are compared with those of Burnett equations and DSMC results by Fiscko and Chapman. The

profiles of the inverse density thickness as a function of the Mach number for Maxwellian gas, hard-sphere gas and Argon gas model have been obtained. The results for the augmented Burnett equations agree well with those of Fiscko and Chapman for the Burnett equations (Fig. 2).

At the same time, the stability characteristics of the different equations are tested for a Mach 20 shock by numerical computations with progressive refinement of the computational meshes. It is found that as the grid meshes are refined by increasing the mesh number, both the Burnett equations and the Burnett(-) equations [6] yield solutions when mesh number is small, but when the number is lager than a certain maximum value, the computations become unstable and break down. In contrast, the augmented Burnett equations are stable for all the mesh numbers tested up to the limit of 1200 investigated. The maximum numbers of mesh points for computational stability are:

Equations	maximum mesh points
Burnett	87
Burnett(-)	209
A-Burnett	> 1200

The Applications of the Augmented Equations to Two Dimensional Problems

The next step of the present research is the application of the Burnett equations to two dimensional problems. In their 1975 paper, Tannehill and Eisler [13] used the Burnett equations in conjunction with Schamberg's second order slip boundary conditions [11] to compute the hypersonic leading edge flow over a flat plate. The results deviated substantially from those of either experiments or the Navier-Stokes equations. Now, the same problem is being reinvestigated in the present investigation. Both the two-dimensional Burnett equations and the augmented Burnett equations are being computed in the flow field, using the implicit Gauss-Seidel iteration method similar to the one proposed by MacCormack for the Navier-Stokes equations [9]. The preliminary results show that the poor results of Eisler and Tannehill's computations may be caused by the Schamberg's boundary conditions. Research in this direction is underway, and it is expected that the results will be reported by the time of the conference next summer.

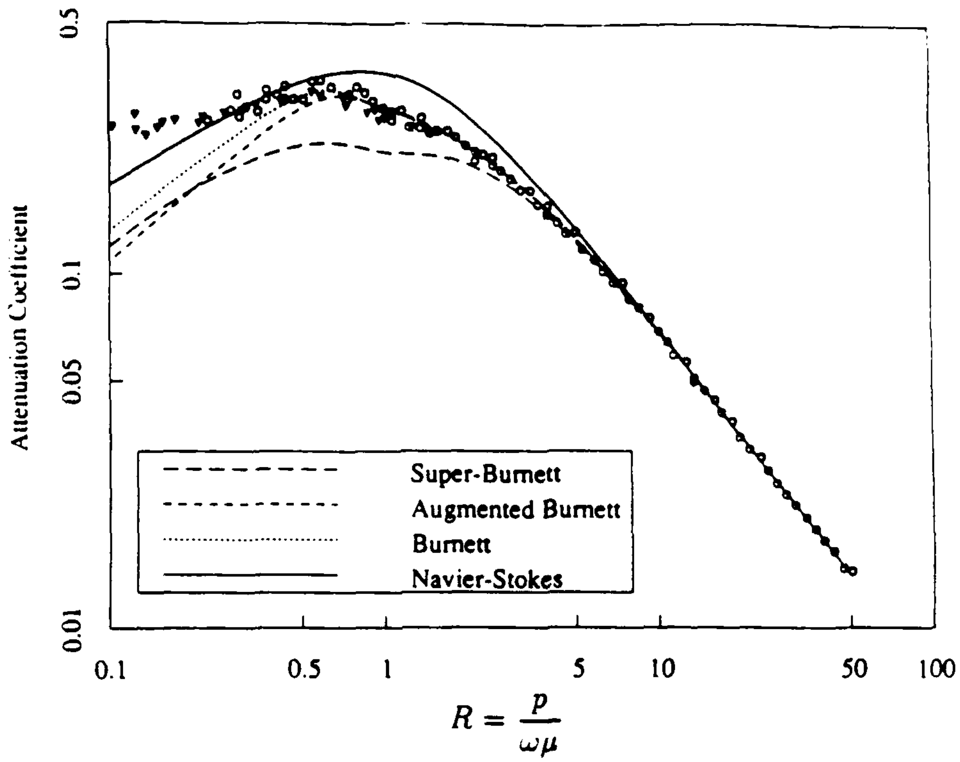

Figure 1. Attenuation coefficients of sound wave in monatomic gases. Experimental measurements: circles–Greenspan [8]; triangles–Meyer & Sessler [10].

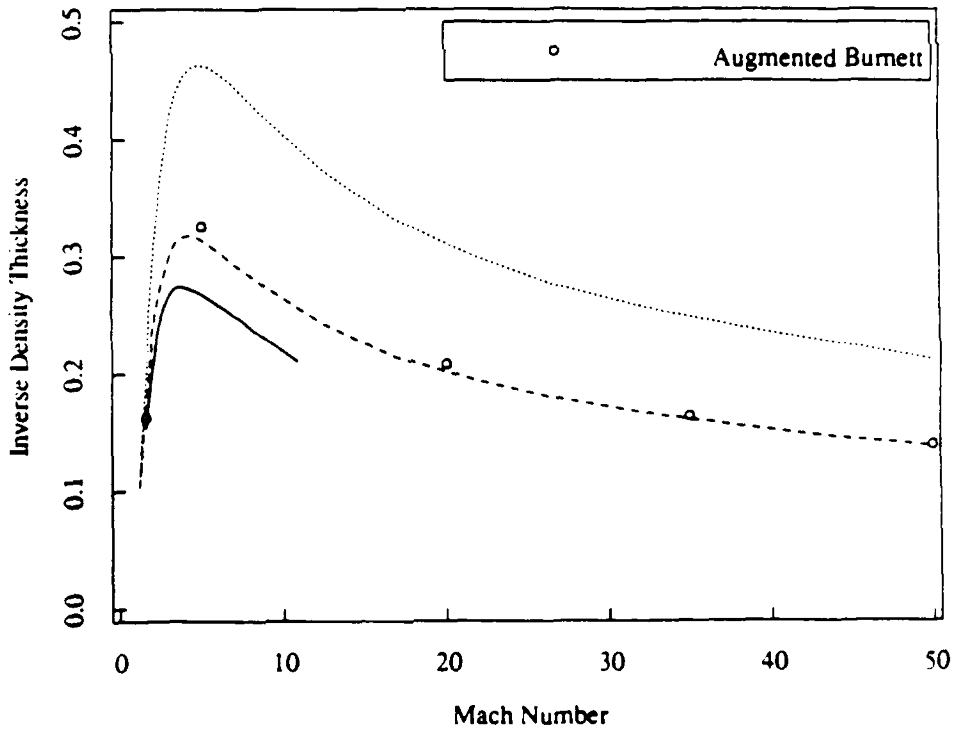

Figure 2. The comparison of the results of the argon shock wave inverse density thickness for the augmented Burnett equations with: The results of Fiscko & Chapman (dash line–Burnett, dotted line–Navier-Stokes); The results of experiments [1](solid line).

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